Sweet Mirage

Adam Benslama

Princeton High School

March 07, 2021

Abstract

A simple experiment is presented to replicate the Fata Morgana effect in the laboratory. A quantitative analysis is done using ray tracing with both photographic and mathematical techniques. The mirage's image, as seen by the eye or the camera lens, can be used to analyze the bending of light rays as well as the distortion of the mirage's image.

Introduction

A mirage is a marvelous natural optical phenomenon. Thermal gradients in the atmosphere and the associated refractive index changes make light rays bend. An object can be seen in a higher or lower position than it actually is, creating an optical illusion. For example, on a hot summer day, one may perceive what appears to be puddles of water on a hot distant road, on which we perceive what appears to be the reflections of other objects. Another example is the Fata Morgana effect [1] which is a type of superior mirage by which distant objects appear inverted, distorted, displaced or multiplied. This type of mirage typically occurs above a cool surface on a hot day.

Mirages offer an excellent opportunity to address several aspects of optics, such as light propagation, the reversibility of light rays, the refractive index and the physical realization of an optical image.

A mirage can be reproduced in the laboratory using different techniques. Those that use air, to simulate the atmosphere, require a specific heated surface [2]. However, a mirage effect can also be achieved by using water at different temperatures [3], or mixable liquids of different densities [4] [5]. The key point for these types of experiments to be effective is to achieve a stable enough refractive index gradient. As a consequence, a well defined pattern of light rays will be produced.

In this paper, we present a simple way to replicate the Fanta Morgana effect in the laboratory. Our setup shows a detailed structure of the mirage with the corresponding behavior of light rays to be observed. We also discuss a ray tracing procedure we used to qualitatively reconstruct the mirage. We used a camera with a known field of view and a transparent container to consistently take pictures of the container from the same distance. The container took up the entire field of view of the camera.

Theory

A superior mirage takes place when an object is seen in a position higher than its actual one. The light rays are bent downwards because of the decreasing refractive index as a function of height. As a consequence, the object is seen hanging over the real one. The Fata Morgana effect, studied here, is a particular case of a superior mirage. The mirage's image is flattened and moved up because of the decreasing refractive index gradient with height.

The refractive index of air as a function of temperature is conventionally modeled by [6] [7] :

$$n(T,p,e) = 1+10^{-6}\times(77.6 + \frac{p}{T} + 3.75\times10^{5}\times\frac{e}{T^{2}}) \quad (1)$$

where T is the temperature (it varies with height and time), e is the partial pressure of water vapor, p is the dry pressure, and n is the refractive index. The temperature of air at a given height and time can also be modeled using a heat equation [8]:

$$\frac{\partial T(\vec{r},t)}{\partial t} = \alpha \times \Delta T(\vec{r}) \quad (2)$$

Where $\vec{r}$ is the position vector, T is the temperature, and α is the thermal diffusivity.

If we move far away from the cool surface, we can assume that the temperature remains constant. Therefore, at a given height far away from the cool surface, the solution to equation 2, which is the equilibrium temperature, can be modeled as follows:

$$T(h) = \frac{T_0 \; - \; c}{L} \times h \; + \; c \quad (3)$$

Where T is the temperature, h the height at which the temperature is constant and equal to c, T_0 is the original temperature at a height of zero, and L is the horizontal length of the medium.

In order to reconstruct the light ray paths, we followed the same procedure described in reference [4]. We solved a differential equation for the light ray paths using Snell's law, and then we used the refractive index gradient, defined in equation 1, to predict the light ray path from a known initial position and slope of the light ray. The differential equation [9] for obtaining the light path y(x) is

$$y^{''}(x) - \frac{n^{'}(y)}{n(y)} \times (1 + (y^{'}(x)^{2}) = 0$$

where n(y) is the refractive index as a function of height, x is the horizontal coordinate and y represents the vertical coordinate. First derivatives are denoted as $y^{'}$ and second derivatives are denoted as $y^{''}$.

Experiment

During the Fata Morgana's superior mirage, the refractive index of the medium decreases as height increases. In order to replicate this phenomenon in the laboratory, we used a sugar-water solution with a refractive index gradient.

First, we measured the refractive index of the sugar-water solution as a function of the sugar concentration. To perform this study, we filled a transparent recipient with water, then we added the quantity of sugar needed to obtain the desired sugar concentration. The solution was then stirred in order to form a homogenous mixture. A laser was shined through the container at a given angle, then the refracted angle was recorded using a camera with a known field of view. A computer program was used to analyze the images and also to measure both the incident and the refracted angles. Snell's law was then used to compute the refractive index of the solution. This procedure was repeated for different concentration values. Figure 1 shows the experimental setup used to perform these studies, and figure 2 shows the refractive index as a function of the sugar concentration.

In order to replicate the Fata Morgana effect in the laboratory, first we setup the medium in which the effect will occur. For this purpose, we used a transparent container ($18.5cm \times 14cm \times 9cm$) filled with 1.665 liters of water. The sucrose sugar was then added to the water to form a layer of sugar at the bottom of the container. Then we waited 48 h to allow the sugar to naturally diffuse in the water. The resulting sugar water solution contained 20% of sugar and 80% of water. The prepared sugar-

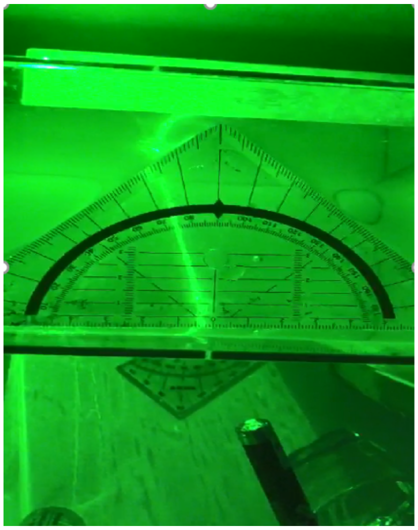

Figure 1: Experimental setup to measure the refractive index for a given concentration

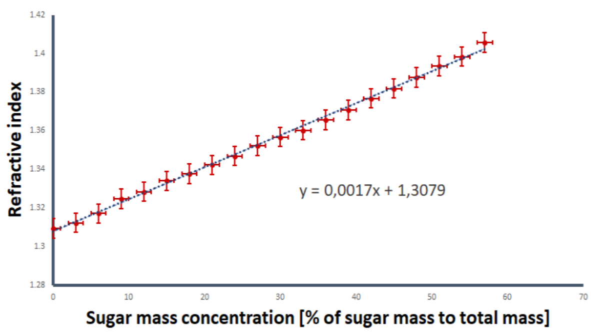

Figure 2: Refractive index as a function of sugar concentration

water solution had a concentration gradient from bottom to top. Figures 3 and 4 summarize the experimental setup used to study the Fata Morgana effect in the Laboratory. Figure 5 shows the refracted trajectory of light through the medium by shining a green laser pointer, with a wavelength of (532 $\pm$ 10) nm, through the container.

Once the medium was setup, using a camera with a known field of view, we took a direct picture of a green object and we compared it to a picture of the same object, taken through the container filled with the sugar-water solution (the container was placed between the camera

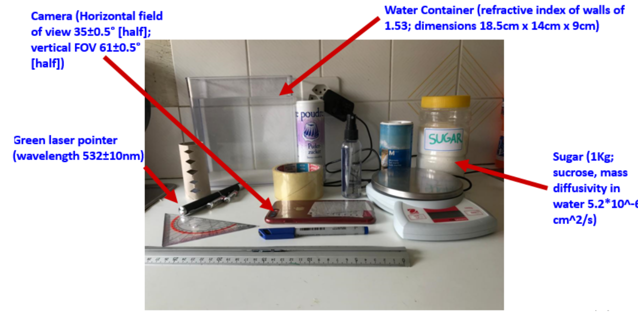

Figure 3: Experimental setup used to study the Fata Morgana effect in the laboratory

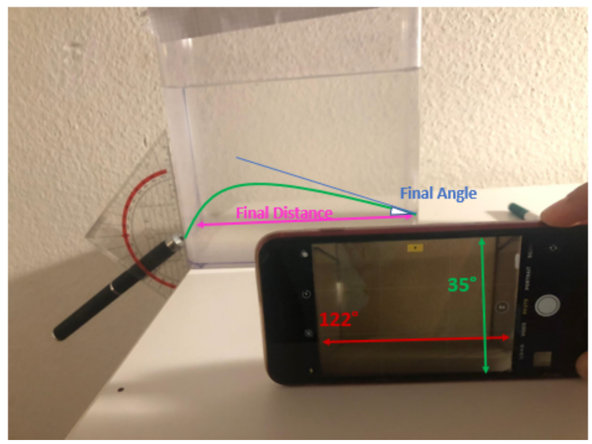

Figure 4: Experimental Setup used to study the Fata Morgana effect in the laboratory

and the object). Figure 6 shows the picture of a green object taken through the medium (sugar-water solution) alongside a picture of the same object taken without the medium.

The angular sizes (heights) of both images were measured. Then a green laser pointer was shined through the container from the same position where the camera was placed. Two angles were recorded, the angle at which the trajectory of light hit the top of the object, and the angle at which the trajectory of light hits the bottom of the objects. Using these two initial angles, we calculated the theoretical trajectories of light. Using these new theoretical values, we predicted the angular height of the object through the illusion and we compared it to the measured value.

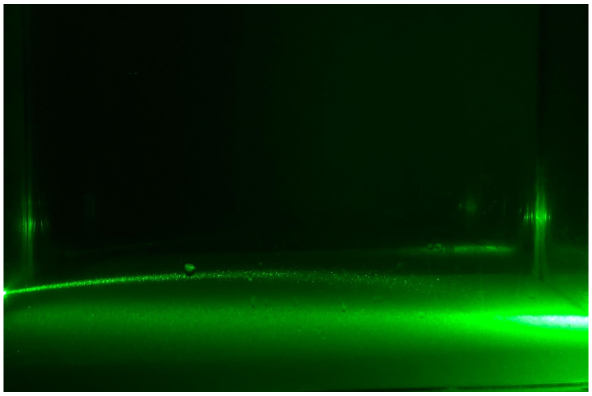

Figure 5: Refracted trajectory of light through a sugar-water solution with a concentration gradient

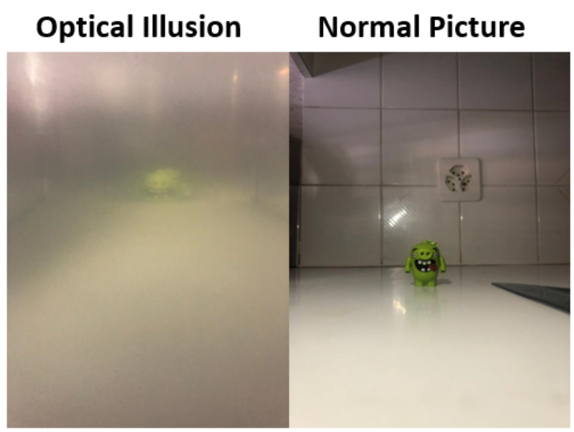

Figure 6: Image of the object taken through the container compared to the object's image without the container

Analysis

The use of gradient refractive index media in the laboratory to illustrate various mathematical problems and physical phenomena is well reported in the literature [4] . In particular, ray tracing in a gradient refractive index media is described by Mamola et al. who solved the differential equation for the light ray paths using Snell's law. They used the experimental refractive index gradient to predict the light ray path from a known initial position and slope of the light ray. We used here a similar approach to compare the actual size of the object with the recorded image of the mirage (as directly recorded by our camera). Our experiment and photographic record of the light rays allows for a quantitative determination of the mirage's image point by point. This gives a good comparison between the predicted (ray-traced) mirage's image and the real image.

First, we measured the refractive index of the medium as a function of height, then we used this value as an input to our theoretical model in order to get the predicted size of the object. Figure 7 shows the measured refractive index as a function of height in our sugar-water solution, and figure 8 shows the comparison between the predicted trajectory of light and the trajectory's picture taken with our camera.

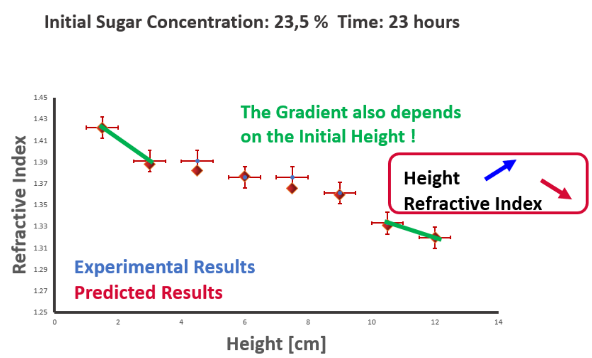

Figure 7: Refractive index gradient as a function of the height in the medium (sugar-water solution)

Taking the ratio between the angular size of the mirage's image and the angular size of the real objet, we found a distortion coefficient of 0.66 ± 0.10. We also used the tracing method in order to calculate the distortion coefficient. To perform this analysis, we measured the angle α_1 at which the light leaves the top of the object and the angle θ_1 at which the light arrives to our eyes. Then we measured the angle α_2 at which the light leaves the bottom of the object and the angle θ_2 at which the light arrives to our eyes. The ratio between $\theta_1 - \theta_2$ and the angular size of the real objet gives a coefficient distortion value of 0.69. Figure 9 illustrate the tracing method used to measure the distortion coefficient.

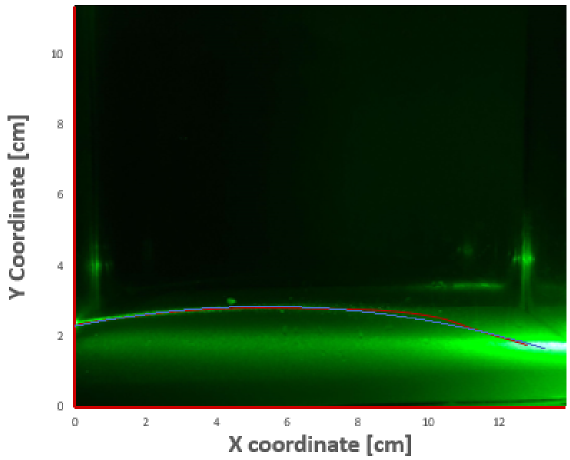

Figure 8: Comparison between the predicted trajectory of light and the trajectory recorded with our camera

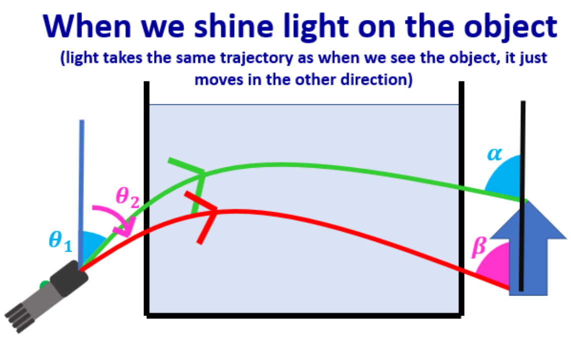

Figure 9: Explanation of the tracing method used to measure the distortion coefficient

Conclusion

The Fata Morgana's gradient index caused by temperature changes in the air was replicated in the laboratory by using a gradient index caused by a sugar-water solution with a concentration gradient. We modeled the trajectory of light using a differential equation and this trajectory has been used to predict the shape of the optical illusions.

References

[1] Blanco-García, J and Vázquez Dorrío, José and Ribas, Fernando, "PHOTOGRAPHING MIRAGES ABOVE THE SEA SURFACE", 8th International Conference on Hands-on Science. At: Ljubljana (Eslovenia), (2011)

[2] E. Fabri, G. Fiorio, F. Lazzeri and P. Violino, "Mirage in the Laboratory", *Am, J. Phys.* ***50 517*** *(1982).*

[3] R. Barker, P.R.M Crofts, and M. Gal, "A superior 'superior' mirage", *J. Am. Phys.* ***57 953*** *(1989)*

[4] K.C. Manola, W.F. Mueller, and B. Regittko, "Light rays in gradient index media: a laboatory exercise", *Am, J. Phys.* ***60 527*** *(1992)*

[5] R.G Greenler, "Laboratory simulation of inferior and superior mirage", *J. Opt. Soc. Am. A* ***4 589*** *(1987)*

[6] J.G. Speight, "Properties of Inorganic Compounds", *Environmental Inorganic Chemistry. Elsevier, (2017)*

[7] National Institute of Standards and Technology (NIST), Engineering Metrology Toolbox, `https://emtoolbox.nist.gov/wavelength/EquationA49.asp`

[8] Heat Equation, Wikipedia, The Free Encyclopedia, `https://en.wikipedia.org/wiki/Heat_equation`

$Online$; $accessed\ febraury$, 16, 2021

[9] T. Lopez, G. Calza, L.M. Gratton, and S. Oss, "Mirages in a bottle", *IOP (2009)*